A Bending Beam Approach for Capturing Ejection Shocks on Missiles

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ABSTRACT

*Ejectors on modern fighter aircraft are designed to provide very high accelerations to the ejected missiles in order to ensure a wide subsonic and even supersonic safe release envelope. Due to the very short duration of these high loads, which rarely exceed 70 milliseconds, the ejection process can be considered as a shock vertically impacting the exposed missile body causing local normal accelerations of the elastic missile being well over 35 g. For a critical assessment of the forces and accelerations of such processes as well as the structural responses a novel computational tool for the time accurate simulation of the ejection of slender bodies from A/C has been realized. It is a typical multidisciplinary interaction problem, which combines the force characteristics of the ejection unit with the instantaneous elastic geometry and position of the missile relative to the ejector pistons. Concerning the unsteady elastic deformation of slender missiles the Bernoulli-Euler theory for bending beams proved to be adequate. For realistic elastic simulations, however, this theory had to be extended for dynamic damping and Coulomb type material friction. A decomposition of the fifth order damped bending beam equation to a system of simpler parabolic/elliptic equations turned out to be a more powerful procedure than solving the original equation directly. A minimum residual method similar to GMRES, however based on a matrix free **full** diagonalization of the minimum square problem has been adopted for the efficient numerical solution of the very stiff system of the algebraic equations.*

1.0 MODELLING OF THE EJECTION PROCESS

The ejection is modelled by two numerical procedures. The first one provides a description of the ejection characteristics either by a preprocessed and tabulated acceleration data file or by running simultaneously a driving gas routine. For each given position of (in our case) two pistons, which push the missile vertically downwards, the code returns the two piston forces. These two forces are input to a second code, which computes the structural response of the missile. In this code the missile motion together with its deformations is integrated by a time stepping procedure. The new positions and deformations of the missile are input for the next call of the piston driving routine. A graphical representation of this interplay is given in fig.1.

In this paper particular attention is paid to the structural response simulation of the missile. Since the time step of the structural response is limited to very small values, typically in the magnitude 10^{-6} sec, for numerical stability and sufficient time resolution, calling the piston response after each time step is prohibitive because of exorbitant computer time connected to such a process. Therefore the piston routine is called after running a series of some 1000 time steps with the structural scheme, during which the forces are approximated by either constancy (Euler forward) or by linear interpolation (predictor-corrector

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scheme). For practical implementation the first approach seems to be sufficient being aware that considerable empiricism is included in the model elsewhere. The geometry for the piston drive input are time and mass averaged over the period between two calls such that the center of gravity is recovered as well as the proper moment of mass inertia.

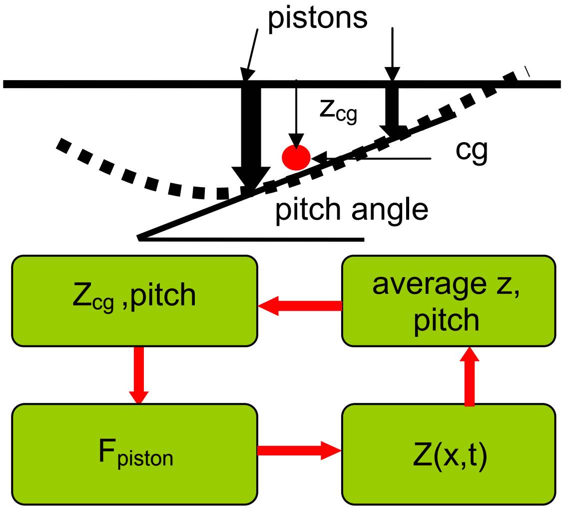

Figure 1: Schematic of ejection process simulation

2.0 MATHEMATICAL FORMULATION

2.1 Bernoulli-Euler Bending Beam Equation for Plane Deformation

The classic bending beam equation combines the stiffness properties of a beam with the dynamics caused by outer forces acting on the beam. A slender missile with moderate cross sectional area variations may be well represented provided the assumptions of linear spring characteristics and plane cross sections normal to the geometric center-of-gravity-line do apply. Longitudinal forces will be neglected:

$$\rho A\left(\ddot{z}+g\right)+(EIz'')''+p=0$$

E is Young's modulus, I is the geometric cross sectional moment of inertia, z is the vertical beam elevation (mathematical convention, upwards positive), g is the gravitational acceleration, ρ is the material density, A is the cross section area, p is a running outer force. Dots mean time derivatives, dashes mean geometric derivatives in x-(longitudinal) direction

2.2 Extension of the Bending Beam Equation for Dynamic Damping

The Bernoulli-Euler theory is based on the assumption that the stress increment $d\sigma$ along a longitudinal fiber element dx is proportional to its length change $d(dx)$:

$$d\sigma=\sigma\left(x+dx\right)-\sigma\left(x\right)=E\frac{d\left(dx\right)}{dx}$$

If unsteady deformations are accounted for, the accommodation of the fibres to the new equilibrium state takes some short time, which may be called a relaxation time. It may be expressed as a first order Taylor expansion of the length change with respect to time:

$$d\sigma = E \frac{d(dx) + \tau \partial \frac{d(dx)}{\partial t}}{dx}$$

τ is of the dimension of time and will be taken herein as an empirical material property.

2.3 Extension of the Bending Beam Equation for Coulomb Friction

The friction caused by the shear between the fibres and in particular the fixing of the different parts of the real body is taken to be Coulomb type. Thus the stress increment should be extended like in spring theory:

$$\dot{\xi} = \partial \frac{d(dx)}{\partial t} \qquad d\sigma = E \frac{d(dx) + \tau \dot{\xi} + f\, sign(\dot{\xi})}{dx}$$

f is a further material constant of the dimension of a length. In order to retain differentiability the Coulomb friction term is slightly modified:

$$f\, sign(\dot{\xi}) \quad \Rightarrow \quad f \frac{\dot{\xi}}{|\dot{\xi}| + \varepsilon}$$

ε controls the slope of the friction term at vanishing deformation speed. A summary of the spring model is given in fig. 2.

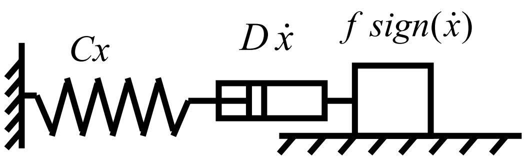

Figure 2: Spring model used for the bending beam equation

2.4 The Extended Bending Beam Equation

After the transformation of the geometric relations to the original equation for expressing the local normal stress one obtains the bending beam equation with damping and friction included.

$$\rho A(\ddot{z} + g) + \{EI[z'' + \tau \dot{z}'' + f\, \dot{z}''(|\dot{z}''| + \varepsilon)^{-1}]\}'' + p = 0$$

2.5 Boundary Conditions

2.5.1 Global Boundary Conditions

Since the end points of the missile are not suspended, bending moments and transversal forces are zero at these places. Therefore second and third x-derivatives of the elevation at the end points of the beam should vanish.

2.5.2 Imposing the Yoke/Piston Forces on the Beam

Imposing vertical discrete forces as boundary conditions at arbitrary *x*-stations is very hard because it would require the solution of accompanying integral equations. Therefore discrete transversal forces *T* are spread over small intervals of *x* using the definition of the running force *p*, which is zero everywhere except at the places of the impact forces:

$$T = \int p dx = p_a \Delta x \qquad \Rightarrow \qquad p_a = \frac{T}{\Delta x}$$

3.0 NUMERICAL FORMULATION

3.1 Decomposition of the Bending Beam Equation

To solve the bending beam equation with the material properties of a real missile turned out to be a formidable task. It required several thousands of Gauss-Seidel (GS) iterations per single time step thus leading to exorbitant computer residing times. An important step to less time consuming calculations is the decomposition of the single high order equation to a system of lower order equations. The following concept proved successful:

$$v - (EI\tau\, z)'' = 0\,, \qquad \rho A \dot{z} + K_w w'' + v'' + c = 0$$

$$K_w \dot{w} - EI[z'' + f\,\dot{z}''(|\,\dot{z}''\,| + \varepsilon)^{-1}] + K_c c = 0\,, \qquad \dot{c} - K_c c'' - (p + \rho A g) = 0$$

K_w and K_c are coupling constants. *c,v,w* are artificial coupling functions. By differentiation and subtraction/addition the basic equation is recovered. The viscosity coefficient K_c may be put close to zero.

3.2 Finite Difference Scheme

The derivatives of the functions occurring in the system of equations are formulated with second order accuracy:

$$\varphi'' = 2\frac{\dfrac{\varphi_i - \varphi_{i-1}}{x_i - x_{i-1}} - \dfrac{\varphi_{i+1} - \varphi_i}{x_{i+1} - x_i}}{x_{i+1} - x_{i-1}} \qquad \dot{\varphi} = \frac{3\varphi^{n+1} - 4\varphi^n + \varphi^{n-1}}{2}$$

3.3 Global Numerical Boundary Conditions

function	zero	linear extrapolation
c, w,v	**x**	
z		**x**

Table 1: Numerical BC's at the end points of the beam

3.4 Force Input

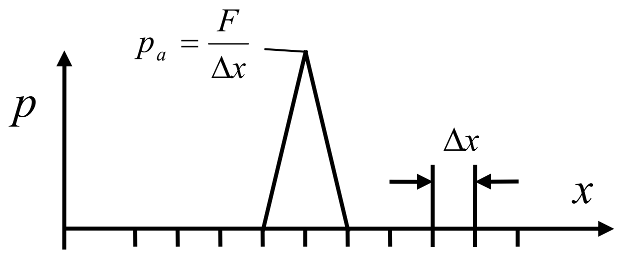

Figure 3: Discrete Force Representation

3.5 Update

3.5.1 Initial guess for a new time step

At the time levels *n* and *n-1* the functions *c,w,z* are known as well as their numerical time derivatives at level *n.* Considerable computer time may be saved if the initial guess for the time level *n+1* is extrapolated from known values by $U^{n+1}_{initial} = U^n + \dot{U}^n \Delta t$.

3.5.2 Gauss-Seidel preconditioning

Approximate solutions for the new time level are found from the residuals *r* of the constituting equations for *c,w,z* by Gauss-Seidel updates

$$r_z = \rho A \dot{z} + K_w w'' + v'' + c$$
$$r_w = K_w \dot{w} - EI[z'' + f\,\dot{z}''(|\,\dot{z}''\,| + \varepsilon)^{-1}] + K_c c$$
$$r_c = \dot{c} - K_c c'' - (p + \rho A g)$$
$$U^{n+\mu+1}_{GS} = U^{n+\mu} - \frac{CFL}{D} r^{\mu,\mu+1}$$

D is the local estimated scalar diagonal element of the equation to be updated and μ is the GS iteration counter. *CFL* is a user specified dimensionless CFL number usually close to unity. Updating is performed by immediate insertion of the newest available values.

3.5.3 Acceleration of the numerical solution

The friction and damping terms contained in the system of equations causes very slow convergence of the GS procedure to a stationary approximate solution per time step. In contrast to the ideal elastic case several thousand iterations may be necessary for just one time step if the coefficients *f* and τ are in the range of values necessary for a realistic simulation. Therefore a means for numerical acceleration has been introduced which is based on some knowledge from the Generalized Minimum Residual method GMRES. For this purpose after a given small number of GS iterations the differences of the functions and the residuals of the newest to their initial values are stored. The template of the acceleration scheme is given in table 2.

$$
\begin{array}{l}
\textit{values known at time levels } n \textit{ and } n-1, \textit{ find solution at } n+1 \\
\textit{initialize}: r_0 = r^n, U_0 = U^n \\
Do\ i = 1, restart \\
\quad Do\ \ j = 1, diagonal \\
\quad\ \textit{run some few GS} - \textit{steps for an approximate solution of } U \\
\quad\ \textit{compute } \Delta U^j = U^{GS} - U_0 \quad \Delta r^j = r^{GS} - r_0 \\
\quad\quad \textit{if } j-1 \textit{ greater than } 0 \textit{ then} \\
\quad\quad\quad Do\ k = 1, j-1 \\
\quad\quad\quad\quad \textit{sums are taken over all discrete residuals}: a = \dfrac{\sum \Delta r^j \Delta r^k}{\sum \Delta r^k \Delta r^k} \\
\quad\quad\quad\quad \textit{full diagonalization}: \ \Delta U^j = \Delta U^j - a\Delta U^k, \Delta r^j = \Delta r^j - a\Delta r^k \\
\quad\quad\quad kend \\
\quad\quad \textit{end if} \\
\quad\quad\quad \textit{initialize for summation}: U^{n+1} = U_0 \\
\quad\quad\quad Do\ k = 1, j \\
\quad\quad\quad\quad \textit{sums are taken over all discrete residuals}: a = \dfrac{\sum r_0 \Delta r^k}{\sum \Delta r^k \Delta r^k} \\
\quad\quad\quad\quad U^{n+1} = U^{n+1} - a\Delta U^k \\
\quad\quad\quad kend \\
\quad\quad jend \\
\quad r_0 = r^{n+1}, U_0 = U^{n+1} \\
iend
\end{array}
$$

Table 2: Template of a fully Diagonalized Minimum Residual Method for a Time Step Solution

4.0 RESULTS

The following calculations were done for a realistic medium range missile. The mesh consisted of 100 equidistant intervals. The time step was 10^{-6} second in all calculations. The GMRES method was run with 4 search directions and 3 restarts. After each 2000 time steps the driving gas routine was called. Typically values of about $\tau \approx 10^{-4}$ sec and $f \approx 10^{-5}$ m for damping and friction were chosen for the comparisons. The comparison of the computed ejection accelerations with realistic data is based on a telemetry script of an ejection from an A/C in flight. Plots are drawn with z downwards being positive.

4.1 Material Properties of the Missile

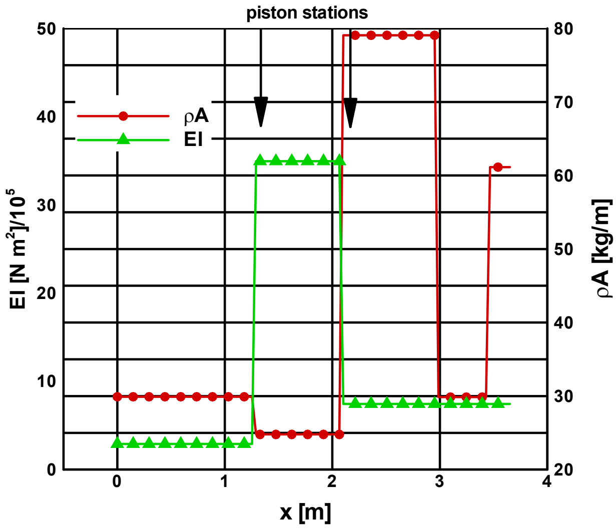

Figure 4: Missile Properties

4.2 Comparison of Rigid Missile versus Elastic Missile Ejection Simulation

4.2.1 Vertical Acceleration of Missile Center of Gravity

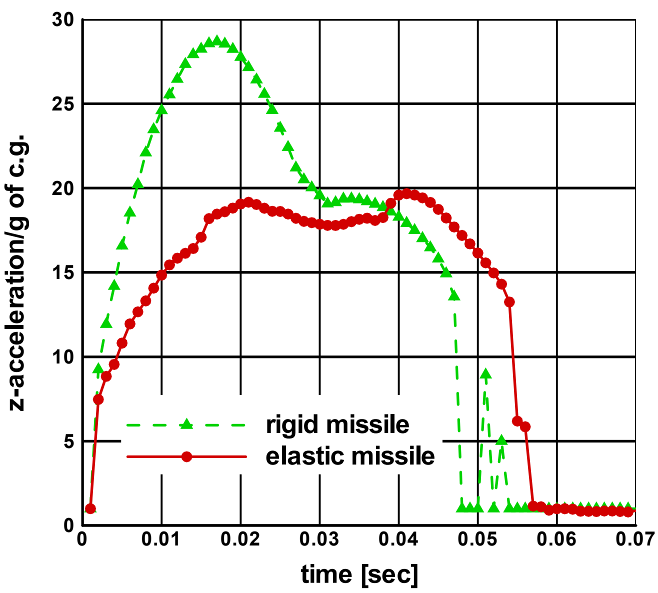

Figure 5: Global z-Acceleration

4.2.2 Vertical Velocity/Displacement of Missile Center of Gravity

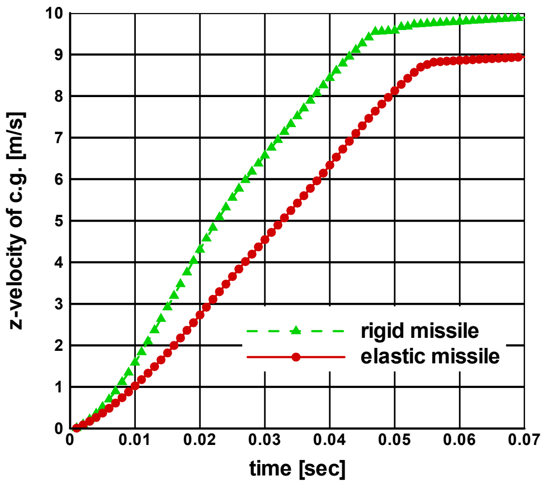

Figure 6: Global Velocity

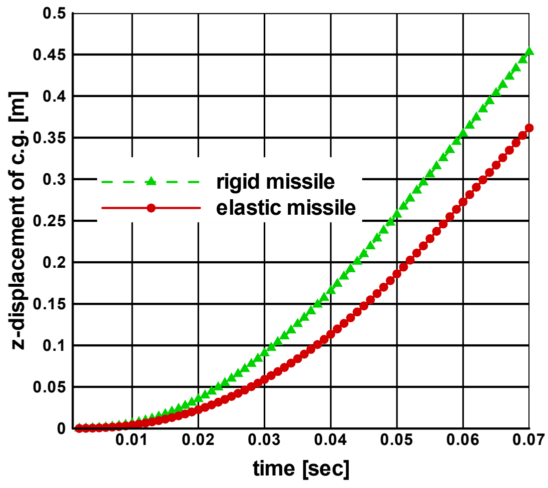

Figure 7: Global Displacement

4.2.3 Forces

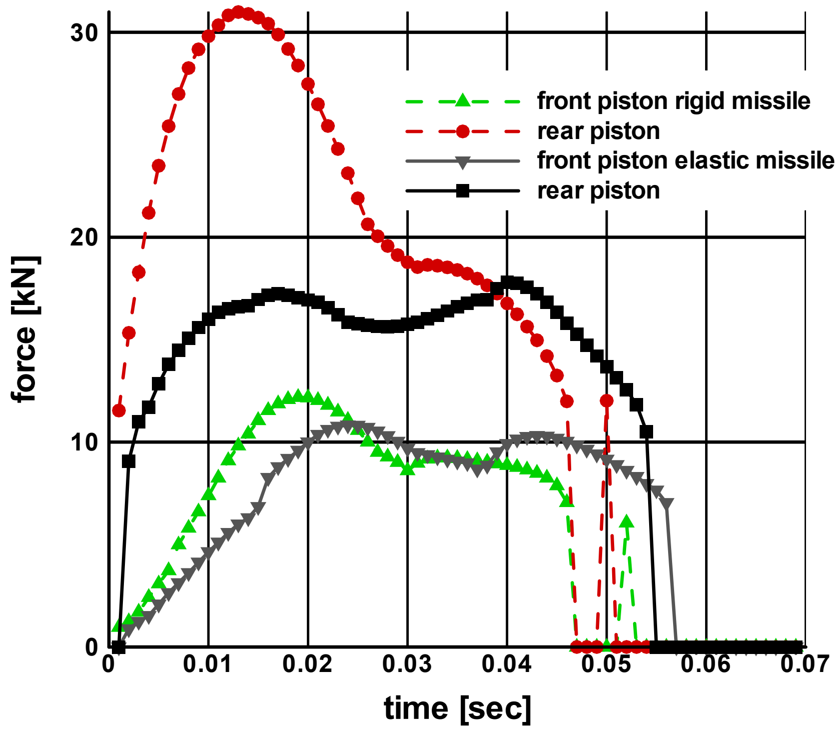

Figure 8: Piston Forces

4.3 Comparison of Flight Test Data versus Elastic Missile Ejection Simulation

The flight test data shown in the following figures were taken from accelerometers placed inside the missile at a longitudinal station close to the forward yoke. See fig. 4, x = 1.4 m.

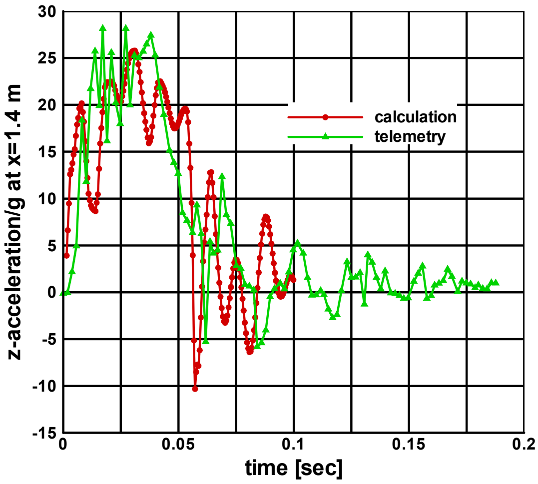

Figure 9: Much Dynamic Damping

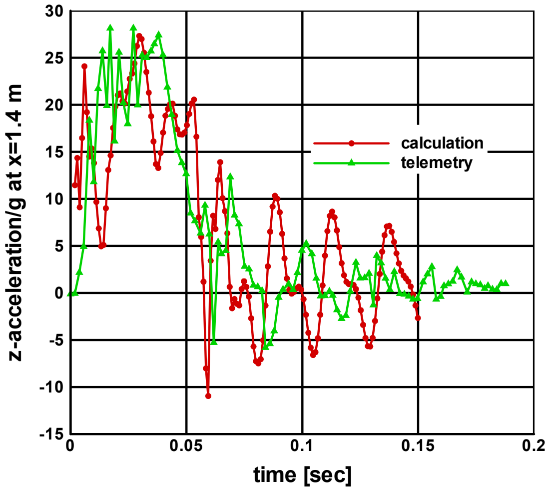

Figure 10: Low Dynamic Damping

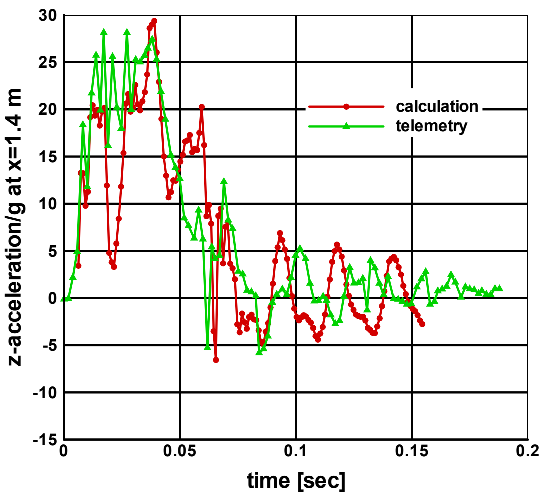

Figure 11: Much Damping + Friction

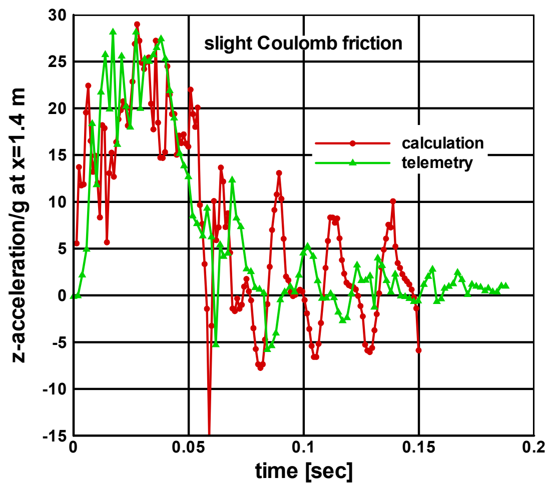

Figure 12: Low Damping + Friction

The end of stroke time is perfectly represented by the calculation, t =.06 sec. Considering the somewhat noisy data from the telemetry, agreement of the calculation with data is acceptable for the ejection phase. The dynamic damping of the calculation, however, does not show the considerable decay of the oscillation amplitudes of the real missile at free flight, $t \geq .06$ sec. Neither a greater parameter for the dynamic damping nor the Coulomb friction turned out to approximate to sufficient agreement both, the ejection phase and simultaneously the start of the free flight.

5.0 CONCLUSION

An innovative approach which is able to predict the time dependent structural response, the deformation and the resulting motion of the missile exposed to ejection shocks was described within this contribution. It was also shown that reciprocal interaction between ejection forces and structural response are essential parts to be modelled.

The results obtained with this scheme prove that simulating the elastic structural response of a missile in spite of using its rigid representation provides an essential contribution to the improvement concerning a more realistic prediction of release processes. As far as the separation process itself is concerned, the ejection performance, eg. end-of-stroke velocity and pitch rate, can drastically change as functionally dependent on the local variation of the missile stiffness. On the other hand the knowledge of these interaction in terms of locally induced acceleration peaks, are not only of paramount importance for the safe separation clearance work but must already be considered for design and development work with respect to the structural limitations.

The lesson learned with the development of this simulation tool comprises the necessity to include damping and friction terms in the constituting equations. Otherwise no realistic simulation of the high-speed impact processes shown herein may be expected.

Another challenge was to provide a fast computing scheme, appropriate for short turn around times for complete trajectories of ejected missiles. On this side of the work presented here, it turned out that the minimum residual methods for solving linear systems are considerably superior over the classic solution methods. A newer version of such a GMRES method is presented herein. It needs no matrix solution, is fully explicit and proved successful also in many CFD computations.

The structural response of a rail launched missile under high maneuvering load factors is also subjected to similar effects, now emerging from the centrifugal and gravitational terms dominating during such flight conditions. There, end of rail shock pulses can reach more than 30 g. In order to capture these effects, the present approach will need further development and expansion work. Especially when dealing with the attempt to describe the interaction between missile, launcher stiffness both as functions of the sliding and releasing characteristics of each missile hanger when disengaging from the rail constraints.

6.0 LITERATURE

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DISCUSSION EDITING

Paper No. 8: A Bending Beam Approach for Capturing Ejection Shocks on Missiles

Authors: Albrecht Eberle, Ronald M. Deslandes

Speaker: A. Eberle

Discussor: Wolfgang Luber

Question: Dynamic damping seems to be very important to tune your results.

-How do you tune the number for the parameter you call relaxation time?

-How do you tune or select the number for the Coulomb Friction?

Speaker's Reply: Without damping local accelerations of several 1000 g did occur with extremely small local amplitudes however. These seem to be sound or noise of no interest for us. I simply chopped the acceleration peaks by tuning on the damping parameters. Luckily a large bandwidth of these 2 parameters provides reasonable results as one may see on the 4 figures with different parameter settings.